**HW - Week 14**

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1. We have six points, each being denoted by pi, where i is their index in the sequence (from 0 to 5). Accordingly we would have 5 As as follows:

Since we have that

Giving m[i,j] = 0 if i=j and

We can construct the m and s tables (i columns, j rows) as follows

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| m | 1 | 2 | 3 | 4 | 5 |
| 1 | 0 | - | - | - | - |
| 2 | 150 | 0 | - | - | - |
| 3 | 330 | 360 | 0 | - | - |
| 4 | 405 | 330 | 180 | 0 | - |
| 5 | 1655 | 2430 | 930 | 3000 | 0 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| s | 1 | 2 | 3 | 4 | 5 |
| 1 | - | - | - | - | - |
| 2 | 1 | - | - | - | - |
| 3 | 2 | 2 | - | - | - |
| 4 | 2 | 2 | 3 | - | - |
| 5 | 4 | 2 | 4 | 4 | - |

Giving the minimum number of required scalar multiplications to be m[1,5] = 1655. Accordingly the optimal solution would be ((A1A2)(A3A4))A5.

1. An optimal substructure can be defined as a situation in which an optimal solution to the problem contains the optimal solutions for its sub-problems. For this problem, the first step would be to split the problem into one or more subproblems, which is acheived by dividing the matrix chain A1A2..An into two smaller sub-chains A1A2...Amand Ak+1A2k+2...An. Accordingly, we could define the optimal solution for each sub-chain to be the most expensive scalar multiplication in the chain. Therefore, the paranthesization in each sub-chain should be chosen in a way to produce the highest cost. This can be proven by the cut and paste method; Let us consider S as the optimal solution for the matrix chain A1A2..An. If the solution for either of the sub-chains is not optimal, S would not be optimal either, which is a contradiction. Hence, this problem exhibits optimal structure.
2. We need to prove that by taking away an, the remaining solution S - {an} is optimal for the following sub-problem KSn-1, w: finding the solution from a1, a2, ..., an with knapsack capacity w. Hence, we suppose that S’ is a different solution from S - {an} and it is optimal to KSn-1, w. Accordingly, if we have that S’ is a feasible solution and also a better solution than S, we would have a contradiction. Initially, we could check the feasiblity of S’ by

which makes S’ a feasible solution as the sum of its weights are lower than or equalt to the capacity w. Accordingly, we would have

since S’ is a better solution than S, which shows a contradiction.